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Technical Report

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Abstract

The induced noise control parameter is defined in terms of the ratio of the stored energy in a master dynamic system, when it is coupled to an adjunct dynamic system, to that stored energy when the coupling is absent. An induced noise control parameter comprises of two factors. The first is defined in terms of the ratio of the indigenous loss factor of the master dynamic system to the virtual loss factor. The virtual loss factor is the sum of the indigenous loss factor of the master dynamic system and the induced loss factor. The induced loss factor measures the influence of an adjunct dynamic system that is coupled to the master dynamic system. The first factor is, then, by definition less than unity. The second factor is defined in terms of the ratio of the external input power into the master oscillator, when coupled, to the external input power in the absence of this coupling. This second factor, which is positive definite, may either exceed unity, be unity, or be less than unity. Both factors in the induced noise control parameter, however, are critically dependent on the global coupling strength. The global coupling strength is again, a ratio. This ratio is that of the energy stored in the adjunct dynamic system and in the coupling to the energy stored in the master dynamic system. The investigation of this ratio and its relationship to the induced

loss factor is explained and graphically illustrated. The modal coupling strength is determined via the global coupling strength. In the statistical energy analysis (SEA) the value of the modal coupling strength, by definition, lies below unity. In the energy analysis (EA) herein developed, the modal coupling strength may exceed unity. This excess occurs in the energy analysis (EA) when the coupling is strong and the damping assigned to the adjunct dynamic system is low. The damping in the adjunct dynamic system is conveniently defined in terms of an associated modal overlap parameter. To reconcile the statistical energy analysis (SEA) with the energy analysis (EA) in focus, the associated modal overlap parameter in the adjunct dynamic system must necessarily exceed a threshold.

Contents:

I. Introduction

Damping treatment is often employed in order to diminish the response of an externally force-driven reverberant dynamic system. The statement is linked to the definition of damping in terms of the loss factor $\eta_o(\omega)$; namely

$$\eta_o(\omega)=\left[\Pi_e^o(\omega)/\omega E_o^o(\omega)\right] \; ; \; E_o^o(\omega)=\left[\Pi_e^o(\omega)/\omega\eta_o(\omega)\right] \qquad ,(1)$$

where $\Pi_e^o(\omega)$ is the power input from the external force-drive, $E_o^o(\omega)$ is the stored energy in the dynamic system and (ω) is the frequency variable [1-3]. The stored energy in a dynamic system is an eligible measure of its (quadratic average) response; see Fig. 1. From Eq. (1) it follows that increasing the damping, which entails an increase in the loss factor, may result in a decrease in the response.

Utilizing Eq. (1), a stored energy reduction scheme may be proposed: The dynamic system in focus - the *master* dynamic system - is modified; e.g., by appropriately coupling it to another; see Fig. 2. The *adjunct* dynamic system, to which the master dynamic system is to be coupled, is suitably designed to change the loss factor as perceived by the external force-drive,

from $\eta_o(\omega)$ to $\eta_v(\omega)$ with $[\eta_o(\omega)/\eta_v(\omega)]<1$. The change, by definition and in the vein of Eq. (1), yields

$$\eta_v(\omega)=[\Pi_e(\omega)/\omega E_o(\omega)]\,;\quad E_o(\omega)=[\Pi_e(\omega)/\omega\eta_v(\omega)] \qquad ,(2)$$

where the corresponding changes in the stored energy and in the external power are from $E_o^o(\omega)$ to $E_o(\omega)$ and from $\Pi_e^o(\omega)$ to $\Pi_e(\omega)$, respectively [1-10]. The external force-drive in this consideration is assumed to remain unchanged under the modification to the dynamic system as just proposed. The *induced* noise control parameter; designated $\xi(\omega)$, may then be expressed, from Eqs. (1) and (2), in the form

$$\xi(\omega)=[E_o(\omega)/E_o^o(\omega)]=\pi(\omega)\xi_o(\omega)\quad;\quad \xi_o(\omega)=[\eta_o(\omega)/\eta_v(\omega)] \qquad ,(3)$$

where the external input power ratio $\pi(\omega)$ is defined

$$\pi(\omega)=[\Pi_e(\omega)/\Pi_e^o(\omega)] \qquad .(4)$$

Employing Eqs. (3) and (4) one may speculate that if the external input power is uninfluenced by the modification; i.e., if

$$\pi(\omega) \equiv 1 \qquad , (5a.1)$$

and if, in addition, the modification successfully achieves a loss factor $\eta_v(\omega)$ such that

$$\xi_o(\omega) = [\eta_o(\omega)/\eta_v(\omega)] << 1 \qquad , (5b)$$

the induced noise control is highly beneficial; namely, the noise control parameter $\xi(\omega)$ is small compared with unity; $\xi(\omega)<<1$. The condition stated in Eq. (5a.1), however, may not hold. After all, a change in the loss factor, perceived by an external force-drive in a dynamic system, may, under certain conditions, be expected to change the external input power. Then, increasing the degree of damping, in the manner here prescribed and stated in Eq. (5b), may result in an input power ratio $\pi(\omega)$ which may either stay as stated in Eq. (5a.1), exceed unity or fall below unity

$$\pi(\omega) > 1 \qquad , (5a.2)$$

or

$$\pi(\omega) < 1 \qquad . (5a.3)$$

In the absence of heavy handedness in the construction of an adjunct dynamic system $\pi(\omega)$ usually exceeds unity; i.e., $\pi(\omega)$ is

usually commensurate with Eq. (5a.2). Then the highly beneficial noise control that Eqs. (5a.1) and (5b) are promising, is mollified as

$$\xi(\omega) = \pi(\omega)\,\xi_o(\omega) > [\eta_o(\omega)/\eta_v(\omega) = \xi_o(\omega) \quad , (5c)$$

where use is made of Eq. (3) [11]. The Principle that underlines this mollifying influence - the mitigator of many a noise control panacea - is attributed to Chatelier [12]. Le Chatelier Principle in Noise Control Engineering is largely the subject matter of this report.

Focusing attention on Fig. 2, the relationships that are governed by the *conservation of energy* are employed to define a number of loss factors, and the global and modal coupling strengths. With the appropriate interpretation of these definitions one may derive an estimation of the external input power ratio $\pi(\omega)$ and, hence, provide a more definitive estimate for this quantity than Eq. (5c) does [11-13]. Then the noise control parameter $\xi(\omega)$, stated in Eq. (3), may be properly estimated. Largely these estimates are cast in terms of mean-values, as prescribed by Skudrzyk [14]. Computational illustrations are cited.

II. Relationships stemming from the conservation of energy

Focusing attention on Fig. 2, one may supplement Eq. (2) with the following definitions derived off the conservation of energy (power). This conservation may be expressed in the forms

$$\Pi_o(\omega) = \eta_o(\omega)[\omega E_o(\omega)]; \quad \Pi_s(\omega) = \eta_I(\omega)[\omega E_o(\omega)] = \eta_s(\omega)[\omega E_s(\omega)];$$
$$\Pi_e(\omega) = \Pi_o(\omega) + \Pi_s(\omega); \quad \Pi_e(\omega) = \eta_v(\omega)[\omega E_o(\omega)] \qquad , (6a)$$

and hence

$$\eta_v(\omega) = \eta_o(\omega) + \eta_I(\omega); \quad \xi_o(\omega) = [1 + \{\eta_I(\omega)/\eta_o(\omega)\}]^{-1};$$
$$\eta_I(\omega) = \eta_s(\omega)\Im_o^s(\omega); \quad \Im_o^s(\omega) = [E_s(\omega)/E_o(\omega)] \qquad , (6b)$$

where $\Pi_o(\omega)$ is the portion of the external input power dissipated in the master dynamic system, and $\Pi_s(\omega)$, $E_s(\omega)$ and $\eta_s(\omega)$ are the portion of the external input power dissipated, the energy stored and the loss factor in the adjunct dynamic system, respectively, and, finally, $\Im_o^s(\omega)$ is the ratio of the energy stored in the adjunct dynamic system to that in the master dynamic system [1-3]. (The coupling elements are considered as part of the adjunct dynamic system; e.g., the stored energy $E_s(\omega)$ includes the stored

energy that resides in the coupling elements.) The stored energy ratio $\Im_o^s(\omega)$ is dubbed the *global* coupling strength between the adjunct and the force-driven master dynamic systems. The loss factor $\eta_I(\omega)$ is dubbed the *induced* loss factor; the loss factor that is induced in the master dynamic system by virtue of its coupling to the adjunct dynamic system [4-13, 15-17]. In the absence of coupling $\eta_I(\omega)$ is identically equal to zero. In addition to the virtual loss factor $\eta_v(\omega)$, defined and stated in Eq. (6), an *effective* loss factor $\eta_e(\omega)$ of the *combined* - - master + adjunct - - dynamic systems may be introduced. This effective loss factor is defined in the form

$$\Pi_e(\omega)=\eta_e(\omega)[\omega E(\omega)]; \quad \eta_e(\omega)=[\Pi_e(\omega)/\omega E(\omega)]; \quad E(\omega)=E_o(\omega)+E_s(\omega);$$

$$[\Pi_e(\omega)/\omega E_o(\omega)]=\eta_v(\omega)=\eta_e(\omega)[1+\Im_o^s(\omega)]; \quad \Im_o^s(\omega)=[E_s(\omega)/E_o(\omega)] \quad . \text{(7a)}$$

From Eq. (7a) the ratio between the virtual loss factor $\eta_v(\omega)$ and the effective loss factor $\eta_e(\omega)$ is simply

$$[\eta_v(\omega)/\eta_e(\omega)]=[1+\Im_o^s(\omega)]\geq 1 \quad , \text{(7b)}$$

where the ratio $\mathfrak{I}_o^s(\omega)$ is explicitly expressed in Eq. (6b). It transpires that the external input power $\Pi_e(\omega)$ into a dynamic system may be cast in the form

$$\Pi_e(\omega) = S_f(\omega) G(\omega) \; ; \; G(\omega) = (\pi/2)[\nu(\omega)/M] \; ; \; S_f(\omega) = \Delta\omega \, s_f(\omega) \qquad , (8)$$

where $s_f(\omega)$ is the power spectral density of the external force drive, $\nu(\omega)$ is the perceived modal density and (M) is the perceived mass of the dynamic system. Finally, $(\Delta\omega)$ is a suitable frequency bandwidth *centered* about a suitable frequency (ω_o) [3]. The perceived quantities are those perceived by the force-drive. From Eq. (8) the external input power $\Pi_e^o(\omega)$, into the master dynamic system in the absence of coupling to the adjunct dynamic system, may then be estimated in the form

$$\Pi_e^o(\omega) \cong S_f(\omega)(\pi/2)[\nu_o(\omega)/M_o] \qquad , (9a)$$

where $\nu_o(\omega)$ and (M_o) are the modal density and mass of the master dynamic system, respectively. The modal density $\nu_o(\omega)$ and the mass (M_o), perceived by the force drive in the absence of

coupling, may be modified by the presence of the coupling. The modified modal density and the mass are designated by $\nu_o^s(\omega)$ and $M_o^s(\omega)$, respectively. It is to be assumed that the modification is related to the averaged modal coupling strength $\varsigma_o^s(\omega)$ in the form

$$\nu_o^s(\omega) = [\nu_o(\omega) + \varsigma_o^s(\omega)\nu_s(\omega)] = \nu_o(\omega)[1 + \mathfrak{I}_o^s(\omega)] \quad , (10a)$$

$$M_o^s(\omega) = M_o[1 + F\{(M_s/M_o)\,\varsigma_o^s(\omega)\}] = M_o[1 + F\{(m_s/m_o)\,\mathfrak{I}_o^s(\omega)\}] \quad , (10b)$$

where $(m_o)$and(m_s) are the *modal* masses in the master dynamic system and in the adjunct dynamic system, respectively, and it is defined that

$$\mathfrak{I}_o^s(\omega) \qquad \varsigma_o^s(\omega) \quad , (11a)$$

$$= [\nu_s(\omega)/\nu_o(\omega)]$$

$$(M_s/M_o) \qquad (m_s/m_o) \quad . (11b)$$

Clearly, Eq. (10a) is less problematic than Eq. (10b) [3,11]. Indeed, the functional form of (F) is not readily definable unless

$\{(M_s/M_o)\varsigma_o^s(\omega)\}$ is small compared with unity; in this event, $F\{(M_s/M_o)\varsigma_o^s\}$ is comparably small [11].

If the power spectral density $s_f(\omega)$ of the external force-drive remains intact when coupling is introduced, then from Eqs. (8)-(10), one derives for the external input power $\Pi_e(\omega)$ the form

$$\Pi_e(\omega) \cong S_f(\omega)(\pi/2)[\nu_o(\omega)/M_o][1+\Im_o^s(\omega)][1+F\{(m_s/m_o)\Im_o^s(\omega)\}]^{-1};$$
$$S_f(\omega) = \Delta\omega\, s_f(\omega) \qquad .(9b)$$

From Eqs. (3)-(10) one, finally, obtains

$$\xi(\omega) = \pi(\omega)\xi_o(\omega) = [1+\Im_o^s(\omega)]\{[1+\{\eta_s(\omega)/\eta_o(\omega)\}\Im_o^s(\omega)][1+F\{(m_s/m_o)\Im_o^s(\omega)\}]\}^{-1} \qquad ,(12a.1)$$

$$\pi(\omega) = [1+\Im_o^s(\omega)][1+F\{(m_s/m_o)\Im_o^s(\omega)\}]^{-1} \qquad ,(12b.1)$$

$$\xi_o(\omega) = [\eta_o(\omega)/\eta_v(\omega)] = [1+\{\eta_s(\omega)/\eta_o(\omega)\}\Im_o^s(\omega)]^{-1} < 1 \qquad .(12c)$$

Except for situations of special design, in the more usual noise control designs, the modal mass ratio (m_s/m_o) is small enough to render $[(m_s/m_o)\Im_o^s(\omega)]$ small compared with unity even when the stored energy ratio $\Im_o^s(\omega)$ may far exceed unity. The exceptions are found in the design of a light panel (the master dynamic system) that is destined to support heavy electronic components (constituting collectively the adjunct dynamic system) in sections of space crafts [18]. By and large, in subsequent considerations it is to be assumed that $[(m_s/m_o)\Im_o^s(\omega)]$ is small compared with unity and, therefore, in subsequent considerations the factor $[1+F\{(m_s/m_o\Im_o^s(\omega)\}]$ is approximated as equal to unity. In this vein, Eqs. (12a.1) and (12b.1) are approximated in the forms

$$\xi(\omega)=[1+\Im_o^s(\omega)][1+\{\eta_s(\omega)/\eta_o(\omega)\}\Im_o^s(\omega)]^{-1} \quad , (12a.2)$$

$$\pi(\omega)=[\Pi_e(\omega)/\Pi_e^o(\omega)]=[\eta_v(\omega)/\eta_e(\omega)]=[1+\Im_o^s(\omega)]>1 \quad , (12b.2)$$

respectively, where the effective loss factor $\eta_e(\omega)$ is defined in Eq. (7). Under this imposition Eq. (12) already poses a question of significance: May an adjunct dynamic system, that is destined

to be passively coupled to an externally force-driven master dynamic system, be appropriately designed to achieve a credible noise control in that master dynamic system? Three cases are detailed in exemplifying three specific but diverse answers to this question:

Case 1. In this case one assumes a priori that the adjunct dynamic system is merely a *sink:* A sink is a dynamic system that *absorbs* power but *does not store* energy. Thus, in this case, $\Im_o^s(\omega) \Rightarrow 0$. From Eq. (12b), therefore, $\pi(\omega)$ is identically equal to unity

$$\pi(\omega) = 1 \qquad .(13a)$$

It follows that when the adjunct dynamic system is a sink and it is attached to the master dynamic system, the external power injection remains unchanged by this attachment. On the other hand the loss factor (η_s) that characterizes the adjunct dynamic system, which is in this case a sink, yields, from Eq. (12), the induced noise control parameter $\xi(\omega)$ to be

$$\xi(\omega) \Rightarrow \xi_o(\omega) = [\eta_o(\omega)] \; [\eta_o(\omega) + \eta_I(\omega)]^{-1} \; ; \quad \eta_I(\omega) = \eta_s(\omega)\Im_o^s(\omega) \qquad .(13b)$$

Here the induced loss factor $\eta_I(\omega)$ is the loss factor contributed to the master dynamic system by an adjunct dynamic system that is a priori a sink. In this case $\eta_I(\omega)$ is commensurate with the loss factor $\eta_o(\omega)$ that is also assumed a priori to be contributed by an attachment to a sink. From Eq. (13) one needs recognize that if $\eta_I(\omega)$ is to be finite, $\eta_s(\omega)$ cannot be selected arbitrarily small compared with unity.

In the remaining two cases; Case 2 and Case 3, the adjunct dynamic system is not a sink. Indeed, in both cases it is assumed that the global coupling strength $\mathfrak{I}_o^s(\omega)$ can be rendered, by design, high compared with unity; $\mathfrak{I}_o^s(\omega) >> 1$. (Nonetheless, as tacitly assumed, even though $\mathfrak{I}_o^s(\omega) >> 1, [(m_s/m_o)\,\mathfrak{I}_o^s(\omega)] << 1$.)

Case 2. If in addition to rendering $\mathfrak{I}_o^s(\omega) >> 1$, the loss factor $\eta_s(\omega)$ in the adjunct dynamic system is designed to highly exceed the loss factor $\eta_o(\omega)$ that is inherent to the master dynamic system; $[\eta_s(\omega)/\eta_o(\omega)] >> 1$, then from Eq. (12) one obtains

$$\xi(\omega) \cong [\eta_o(\omega)/\eta_s(\omega)] << 1 \qquad , (14a)$$

which describes a beneficial noise control. It needs to be said in this connection that were the ratio $\pi(\omega)$ of the external input power assumed to be equal to unity, as stated in Eq. (5a), the apparent noise control achieved under this (false) assumption, would be even more beneficial than that estimated in Eq. (14a); i.e.,

$$\xi(\omega) \Rightarrow \xi_o(\omega) \cong \left[1 + \{\eta_s(\omega)/\eta_o(\omega)\}\Im_o^s(\omega)\right]^{-1} < \left[\eta_o(\omega)/\eta_s(\omega)\right] << 1 \quad .(14b)$$

Case 3. If in addition to rendering $\Im_o^s(\omega) >> 1$, the global coupling strength $\Im_o^s(\omega)$ can be rendered high enough, such that even if, by design, $[\eta_s(\omega)/\eta_o(\omega)] < 1$, $[\{\eta_s(\omega)/\eta_o(\omega)\}\Im_o^s(\omega)]$ is still in excess of unity, then

$$\xi(\omega) \cong [\eta_o(\omega)/\eta_s(\omega)] > 1 \quad .(15a)$$

Equation (15a) describes a noise control *reversal*. Again, were the ratio $\pi(\omega)$ of the external input power assumed to be equal to unity, as stated in Eq. (5a), the noise control reversal would not emerge; namely, under this (false) assumption

$$\xi(\omega) \Rightarrow \xi_o(\omega) \cong [1 + \{\eta_s(\omega)/\eta_o(\omega)\}\Im_o^s(\omega)]^{-1} < 1 \qquad ,(15b)$$

which is a beneficial noise control, thus, contradicting the estimated noise control reversal quoted in Eq. (15a). (Again it is recalled that in this consideration $[(m_s/m_o)\Im_o^s(\omega)]$ is assumed to be small compared with unity.)

A corollary to cases 2 and 3 follows. Were the adjunct dynamic system a loss factor-wise similar to the master dynamic system, in the sense that $\eta_o(\omega) = \eta_s(\omega)$, one will find that Eqs. (12a) - (12c) assume the forms

$$\xi(\omega) \Rightarrow 1 \qquad ,(16a)$$

$$\pi(\omega) \Rightarrow [1 + \Im_o^s(\omega)] > 1 \qquad ,(16b)$$

$$\xi_o(\omega) = [1 + \Im_o^s(\omega)]^{-1} < 1 \qquad ,(16c)$$

respectively. Thus, when $\eta_o(\omega) = \eta_s(\omega)$, the coupling introduces no noise control benefit; the coupling is neutral.

In addition to the obvious roles played by the indigenous loss factors $\eta_o(\omega)$ and $\eta_s(\omega)$ (and the modal masses (m_o) and (m_s)), in the master and in the adjunct dynamic systems, respectively, in the determination of the noise control parameter $\xi(\omega)$, the crucial role played by the global coupling strength $\Im_o^s(\omega)$ is clear in Eq. (12), especially when the adjunct dynamic system is not merely a sink. One recalls that in a sink $\Im_o^s(\omega) \Rightarrow 0$ and $\eta_s(\omega)\Im_o^s(\omega) \Rightarrow \eta_I(\omega)$. The nature and the composition of a non-zero $\Im_o^s(\omega)$ is, therefore, investigated next.

III. Global and modal coupling strengths and masses

As already intimated in Eqs. (10) and (11), the modal coupling strength $\varsigma_o^s(\omega)$ is related to the global coupling strength $\Im_o^s(\omega)$ in the form

$$\Im_o^s(\omega)=[N_s(\omega)/N_o(\omega)]\,\varsigma_o^s(\omega)\ ;\ \varsigma_o^s(\omega)=[\mathcal{E}_s(\omega)/\mathcal{E}_o(\omega)] \quad ,(17)$$

$$E_o(\omega)=\Delta\omega\mathcal{V}_o(\omega)\mathcal{E}_o(\omega)\ ;\ E_s(\omega)=\Delta\omega\mathcal{V}_s(\omega)\mathcal{E}_s(\omega)\ ;$$
$$N_o(\omega)=\Delta\omega\mathcal{V}_o(\omega)=(\Delta\omega/\omega_o)[\omega_o\mathcal{V}_o(\omega)]\ ;N_s(\omega)=\Delta\omega\mathcal{V}_s(\omega)=(\Delta\omega/\omega_o)[\omega_o\mathcal{V}_s(\omega)] \quad ,(18a)$$

where $\mathcal{V}_o(\omega)$and $\mathcal{E}_o(\omega)$ and $\mathcal{V}_s(\omega)$and $\mathcal{E}_s(\omega)$ are the modal density and the modal stored energy in the master and in the adjunct dynamic systems, respectively, $(\Delta\omega)$ is a suitable frequency bandwidth centered about (ω_o)[1-3]. Averaging a la Skudrzyk is implied in the likes of Eqs. (17) and (18) and in some subsequent equations [14]. [cf. Appendices A and B.] In order to proceed further, it is necessary to define more precisely the modal density $\mathcal{V}_s(\omega)$ of the adjunct dynamic system. For this purpose the distribution $X_r;X_r=(\omega_r/\omega_o)$, is required, where (ω_r) is the modal resonance

frequency of the (r)th mode; the modes are assumed to be sequentially indexed. If the index (r) is given a continuous connotation then the continuity and sequentiality of the distribution may be defined in the form

$$X(r) = [\omega(r)/\omega_o] \; ; \; X(r+\varepsilon) > X(r-\varepsilon) \; ; \; \varepsilon > 0 \quad .(18b)$$

One may then define the corresponding, local modal density in the form

$$[\omega_o \nu(r)] = [\partial X(r)/\partial r]^{-1} \quad .(18c)$$

With this definition in place, mean-value estimates of the induced loss factor $\eta_I(\omega)$ may be determined [3, 13, 15, 16]. The result is

$$\eta_I(\omega) \cong (\pi/2)(\omega/\omega_o)^3 [\omega_o \nu_s(\omega)](m_s/m_o)C(\omega) \quad .(19a)$$

The coupling factor $C(\omega)$, in Eq. (19a), is expressed in terms of the normalized coupling coefficients in the form

$$C(\omega)=[1+\mathrm{m}_c(\omega)]^{-1}\ C_o(\omega)\ ;\ C_o(\omega)=[\ \{\mathrm{m}_c(\omega)+\alpha_c(\omega)\}^2+\{g(\omega)\}^2]\quad ,(20a)$$

$$\mathrm{m}_c(\omega)=[m_c(\omega)/m_s]\ ;\ \alpha_c(\omega)=[k_c(\omega)/(\omega^2 m_s)]\ ;\ g(\omega)=[G(\omega)/\omega m_s]\ .(20b)$$

In Eq. (20) the vector $\{m_c(\omega), k_c(\omega), G(\omega)\}$ defines the mass, the stiffness and the gyroscopic coupling coefficients, respectively. Conveniently, one may categorize the coupling factor $C(\omega)$ from-strong-to-weak in the form [13, 15]

$$C(\omega)\cong \begin{cases} (1/\pi);\ \text{strong couplings} & ,(21a) \\ (\pi 10^{-2});\ \text{moderate couplings} & ,(21b) \\ (10^{-3});\ \text{weak couplings} & .(21c) \end{cases}$$

One may utilize the relationships between the *global* masses (M_o) and (M_s) to the corresponding modal masses (m_o) and (m_s); namely

$$[N_s(\omega)m_s/N_o(\omega)m_o]=(M_s/M_o)\ ;\ M_o=N_o(\omega)m_o\ ;\ M_s=N_s(\omega)m_s\quad ,(22)$$

and Eq. (18a) to recast Eq. (19a) in the form

$$\eta_I(\omega) \cong (\pi/2)(\omega/\omega_o)^3[\omega_o \nu_o(\omega)](M_s/M_o)C(\omega) \quad .(19b)$$

Again, it is emphasized that $\eta_I(\omega)$ is the induced loss factor that defines the net power flow $\Pi_s(\omega)$ from the master dynamic system to the adjunct dynamic system in terms of the stored energy $E_o(\omega)$ in the former; i.e., Eq. (6a), in part states

$$\Pi_s(\omega) = \eta_I(\omega)[\omega E_o(\omega)] \quad .(23)$$

Now that the induced loss factor $\eta_I(\omega)$ has been defined and stated in some detail, attention is to be focused on the definition and statement of the modal coupling strength $\varsigma_o^s(\omega)$. From Eqs. (6b) and (17) one finds that to derive $\varsigma_o^s(\omega)$ off $\eta_I(\omega)$ one needs define the indigenous loss factor $\eta_s(\omega)$ in the adjunct dynamic system [13, 15, 16]. For this purpose it is convenient to express $\eta_s(\omega)$ in terms of the modal overlap parameter $b_s(\omega)$ [13, 15,16]. The expression is

$$\eta_s(\omega) = b_s(\omega)[\omega \nu_s(\omega)]^{-1} = b_s(\omega)[\omega_o \nu_s(\omega)]^{-1}(\omega_o/\omega) \quad .(24)$$

where the modal density $\nu_s(\omega)$ conforms to its definition in Eq. (18c). The convenience stems from the role that is played by $b_s(\omega)$ in relating frequency widths of modes to the frequency separation between adjacent modes [3]. In that sense $b_s(\omega)$ determines the degree of damping that $\eta_s(\omega)$ represents; namely

$$b_s(\omega) \quad \begin{array}{l} \ll 1, \text{ light damping} \\ \cong 1, \text{ moderate damping} \\ \gg 1, \text{ heavy damping} \end{array} \qquad .(25)$$

Indeed, although the values of $\eta_I(\omega)$ stated in Eq. (19) appear to be independent of the modal overlap parameter, this independence is not free. The derivation of $\eta_I(\omega)$ in the form stated in Eq. (19) is predicated on a priori assigning a continuous connotation to the indices that identify the modes in the adjunct and/or the master dynamic systems. The assignment allows the summations over modes in the evaluations of the induce loss factor to be replaced by integrations over the continuous indices. It transpires that this process yields average (a la Skudrzyk) values for $\eta_I(\omega)$ when $b_s(\omega)$ is less than unity. (The summations in this instance; i.e., when $b_s(\omega)$ is less than unity, yield induce loss factors that undulate as functions of the normalized frequency

(ω/ω_o). The excusions of the undulations from the mean-values are the more pronounced the lower are the values of $b_s(\omega)$. [Appendix A and B.]) On the other hand, when $b_s(\omega)$ exceeds unity, the evaluation of the induced loss factors by summations over the individual modes substantially match those values derived by integrations over the continuous indices. In this sense $b_s(\omega)$ plays a pivotal role in the interpretation of the data yielded in the subsequent developments of this thesis. From Eqs. (6b), (17), (19) and (24) one obtains

$$\varsigma_o^s(\omega) \cong (\pi/2)(\omega/\omega_o)^4[\omega_o \nu_s(\omega)]^2(m_s/m_o)[C(\omega)/b_s(\omega)] \quad , (26a)$$

$$\varsigma_o^s(\omega) \cong (\pi/2)(\omega/\omega_o)^4[\omega_o \nu_o(\omega)\omega_o \nu_s(\omega)](M_s/M_o)[C(\omega)/b_s(\omega)] \quad , (26b)$$

where

$$[\nu_s(\omega)/\nu_o(\omega)] = [N_s(\omega)/N_o(\omega)] \quad . (27)$$

The induced loss factor $\eta_I(\omega)$, stated in Eq. (19), and the modal coupling strength $\varsigma_o^s(\omega)$, stated in Eq. (26), are exemplified for the complex dynamic system sketched in Fig. 3. This complex

dynamic system comprises a simple harmonic oscillator for the master dynamic system and a number of simple harmonic oscillators that collectively constitute the adjunct dynamic system. The coupling elements are uniformly composed of individual or combined mass, stiffness and gyroscopic components [13, 15, 16]. Only the master dynamic system is externally force-driven; the satellite oscillators, that constitute the adjunct dynamic system, are neither coupled to each other nor externally force-driven. The so described complex dynamic system is amendable for precise definition in terms of the individual oscillators, including the master oscillator, and the individual couplings of the satellite oscillators to the master oscillator. [Appendix B.] Moreover, the so prescribed complex dynamic system facilitates the computations of the designated quantities, $\eta_I(\omega)$ and $\varsigma_o^s(\omega)$, rendering the results of these computations easy to interpret.

IV. Computations of the induced loss factor $\eta_I(\omega)$ and the modal coupling strength $\varsigma_o^s(\omega)$

To facilitate the computations a more definitive statement must be introduced regarding the complex dynamic system. Thus, from Fig. 3 one surmises that $N_o(\omega)$ is equal to unity and, therefore, $m_o \equiv M_o$. [cf. Equation (22).] In addition, the resonance frequency of the master dynamic system in isolation is designated (ω_o). The harmonic oscillators pertaining to the adjunct dynamic system are distinguished, in this case, as satellite oscillators. The number $N_s(\omega)$ of satellite oscillators may exceed unity; $N_s(\omega) \geq 1$. In addition, the resonance frequency of the (r)th satellite oscillator is designated (ω_r). The normalized resonance frequency distribution $X(r)$ of the satellite oscillators may be defined in the form

$$(\omega_r / \omega_o) = [\omega(r) / \omega_o] = X(r) \quad ; \quad 1 \leq r \leq N_s \qquad , (28)$$

where (r) indexes the (r)th satellite oscillator. [cf. Eq. (18b).] The functional form of $X(r)$, as a function of (r), may be

given a continuous connotation; e.g., as exemplified in Fig. 4a. In this figure

$$X(r)=[1+\{1+N_s-2r\}(\gamma/2N_s)]^{-1/2} \quad ; \quad \gamma\cong 0.6 \quad ; \quad N_s(\omega)\geq 1 \qquad ,(29a)$$

and $N_s=27$ [15,16]. Both, the discrete and the continuous, forms of $X(r)$ are depicted in Fig. 4a. [cf. Appendix A.] Also, it is imposed, in this example, that as many satellite oscillators are with resonance frequencies that are less than the resonance frequency (ω_o), as with those that exceed (ω_o) [13]. Indeed, when the number of satellite oscillators is odd, the resonance frequency of one of the satellite oscillators matches that of the master oscillator; for that oscillator, by definition, $r(1+N_s)^{-1}=(1/2)$ and $X(r)=1$ [13]. Similarly, the loss factor η_r, that is associated with the (r)th satellite oscillator, may be cast in the form

$$\eta_r \Rightarrow \eta(r)=b(r)[X(r)]^{-1}[\partial X(r)/\partial r]=b(r)(\gamma/2N_s)[X(r)]^2 \qquad ,(29b)$$

where $b(r)$ is a designated localized modal overlap parameter; localized at and in the vicinity of the resonance frequency (ω_r) of the (r)th satellite oscillator [13]. Employing Eq. (29a),

with $N_s = 27$ in Eq. (29b), $\eta(r)$ is exemplified in Fig. 4b; in this example three constant values of $b(r)$ are used; namely, $b(r)$=(0.1), (2.0) and (10) [15, 16]. Again, both the discrete and the continuous forms of $\eta(r)$ are depicted in Fig. 4b. [cf. Appendix A.]

Using the expression for the normalized resonance frequency distribution $X(r)$, as stated in Eq. (29a), one derives from Eq. (18c), the expressions

$$[\omega_o v_s(\omega)] = N_s(\omega)(2/\gamma)(\omega_o/\omega)^3 \qquad , (30a)$$

$$(\Delta\omega/\omega_o)^{-1} = (2/\gamma)(\omega/\omega_o)^3 = [\omega_o v_o(\omega)]^{-1} \qquad , (30b)$$

where, again, $N_o(\omega) = 1$. From Eqs. (19) and (30) one obtains

$$\eta_I(\omega) = (\pi/2)(2/\gamma)(M_s/M_o)C(\omega) \qquad . (31)$$

The values for the induced loss factor $\eta_I(\omega)$, as a function of the normalized frequency (ω/ω_o), in the appropriate range of

frequency, are depicted in Fig. 5. The normalized range of frequency is given by

$$[\Delta(\omega)/\omega_o] \cong [1-(\gamma/2)]^{-1/2} - [1+(\gamma/2)]^{-1/2} \cong (\gamma/2) \quad . (32)$$

Again, in Fig. 5, $N_s = 27$ and the global mass ratio (M_s/M_o) is set equal to one tenth (1/10). There are three curves in Fig. 5, the solid curve corresponds to a strong coupling with $C(\omega)=1$, the dash curve corresponds to a moderate coupling with $C(\omega)=3x10^{-2}$ and the dash-dot curve corresponds to a weak coupling with $C(\omega)=10^{-3}$. The normalized range of frequency in Fig. 5 is set by $\gamma = 0.6$. Clearly, the induced loss factor increases with an increase in the global mass ratio (M_s/M_o), with an increase in the coupling factor $C(\omega)$ and with a decrease in the normalized range of frequency; namely, a decrease in (γ); notwithstanding that (γ) must be chosen less than unity. It may be of interest to contrast the induced loss factor $\eta_I(\omega)$ with the corresponding assigned values for the indigenous loss factor $\eta_s(\omega)$ in the adjunct dynamic system. The expression for $\eta_s(\omega)$ is derived from Eqs. (24) and (30) to be

$$\eta_s(\omega) = b_s(\omega)(\omega/\omega_o)^2[\gamma/2N_s(\omega)] \quad . (33)$$

The loss factor $\eta_s(\omega)$ is depicted, as a function of (ω/ω_o), for $\gamma = 0.6$ and $N_s(\omega) = 27$, in Fig. 6. The three curves in Fig. 6 pertain to $b_s(\omega) = 0.1$ (solid), $b_s(\omega) = 2.0$ (dash) and $b_s = 10$ (dash-dot). The results in Figs. 5 and 6 may be compared as a bench mark.

Using Eqs. (30) and (33) in Eq. (26) one obtains for the modal coupling strength $\varsigma_o^s(\omega)$ the equivalent expressions

$$\varsigma_o^s(\omega) \cong [\pi/2b_s(\omega)](\omega_o/\omega)^2(2/\gamma)^2[N_s(\omega)](m_s/m_o)[C(\omega)] \quad , (34a)$$

$$\varsigma_o^s(\omega) \cong (\pi/2)(\omega_o/\omega)^2(2/\gamma)^2(M_s/M_o)[C(\omega)/b_s(\omega)] \; ; \; N_o(\omega) = 1 \quad . (34b)$$

The values for the induced loss factor, as a function of the normalized frequency (ω/ω_o), in the appropriate range of frequency, as defined in Eq. (32), are depicted in Fig. 7. Again, in Fig. 7 the global mass ratio is set equal to one-tenth (1/10); $(M_s/M_o) = 0.1$, and the normalized range of frequency is set by $\gamma = 0.6$. Clearly, as is the induced loss factor $\eta_I(\omega)$, the

modal coupling strength $\varsigma_o^s(\omega)$ increases with increase in the global mass ratio (M_s/M_o) and the coupling factor $C(\omega)$. Also, as does $\eta_I(\omega)$, $\varsigma_o^s(\omega)$ increases as (γ) decreases. Significantly, however, the modal coupling strength $\varsigma_o^s(\omega)$ is inversely proportional to the modal overlap parameter $b_s(\omega)$. As Eq. (34b) indicates one may define $[C(\omega)/b_s(\omega)]$ as the modified coupling factor; the modified coupling factor $C^1(\omega)$ exceeds the coupling factor if $b_s(\omega)$ is less than unity and is less than the coupling factor if $b_s(\omega)$ exceeds unity. In each sub-figure of Fig. 7 three curves are depicted; the solid curve pertains to a $b_s(\omega)$ that is equal to one-tenth (1/10), the dash curve pertains to a $b_s(\omega)$ that is equal to two (2) and the dash-dot pertains to a $b_s(\omega)$ that is equal to ten (10). Figure 7a depicts strong coupling with a coupling factor $C(\omega)$ that is equal to unity; $C(\omega)=1$, Fig. 7b depicts moderate coupling with $C(\omega)=3x10^{-2}$ and Fig. 7c depicts weak coupling with $C(\omega)=10^{-3}$. A major observation emerges from Figs. 7a-c; the observation is discussed briefly in the next section.

V. Validity of statistical energy analysis

An observation emerges when the modal coupling strength $\varsigma_o^{sea}(\omega)$ is estimated via the statistical energy analysis (SEA) rather than estimated in the manner described herein; e.g., as stated in Eq. (34) [13]. In terms of SEA the net power $\Pi_s(\omega)$ that invades the adjunct dynamic system is given by

$$\Pi_s(\omega) = \eta_{so}(\omega)[\omega E_o(\omega)] - \eta_{os}(\omega)[\omega E_s(\omega)] ;$$
$$[\eta_{so}(\omega)/\eta_{os}(\omega)] = [N_s(\omega)/N_o(\omega)] = [\nu_s(\omega)/\nu_o(\omega)] \qquad , (35)$$

where $\eta_{so}(\omega)$ and $\eta_{os}(\omega)$ are the coupling loss factors from the master dynamic system to the adjunct dynamic system and vice versa, respectively [3]. From Eq. (35) one obtains

$$[\Pi_s(\omega)/\omega\, E_o(\omega)]^{sea} = \eta_{so}(\omega)[1 - \varsigma_o^{sea}(\omega)]$$
$$\varsigma_o^{sea}(\omega) = \eta_{os}(\omega)[\eta_s(\omega) + \eta_{os}(\omega)]^{-1} \qquad , (36a)$$

or equivalently

$$[\Pi_s(\omega)/\omega E_o(\omega)]^{sea} = \eta_I^{sea}(\omega) = \eta_s(\omega)\eta_{so}(\omega)[\eta_s(\omega)+\eta_{os}(\omega)]^{-1} = \Im_o^{sea}(\omega)\eta_s(\omega);$$

$$\Im_o^{sea}(\omega) = [N_s(\omega)/N_o(\omega)]\varsigma_o^{sea}(\omega) \quad , (36b)$$

where the superscript (*sea*) indicates that the estimates so superscripted and those enclosed in the square brackets are a la SEA [3]. It is clearly evident from Eq. (36) that the modal coupling strength $\varsigma_o^{sea}(\omega)$ must, by definition, lie below unity; namely

$$\varsigma_o^{sea}(\omega) < 1 \quad . (37)$$

Equation (37) is a tenet of the statistical energy analysis (SEA) [3, 19].

For the complex dynamic system depicted in Fig. 3 and in terms of the analysis presented herein this restriction on the modal coupling strength may be stated, from Eqs. (34) and (39), to be

$$b_s(\omega) > [b_s(\omega)]_M = (\pi/2)(\omega_o/\omega)^2(4/\gamma^2)(M_s/M_o)C(\omega) \quad , (38)$$

where $[b_s(\omega)]_M$ designates the minimum value of the modal overlap parameter that is needed to validate the statistical energy analysis. In Fig. 8 the values of $[b_s(\omega)]_M$, as a function of (ω/ω_o), is depicted for three values of the coupling factor $C(\omega)$; $C(\omega)=1$ (dash-dot), $C(\omega)=3x10^{-2}$ (dash) and $C(\omega)=10^{-3}$ (solid). It is observed, in Fig. 8, that (SEA) is validated for values of $b_s(\omega)$ that are less than unity only for coupling factors that are small compared with unity. It is, thus, speculated that the use of (SEA) may be strained when dealing with strong and even with moderate couplings. The validation of (SEA) may be called to question when the loss factors that are associated with the adjunct dynamic systems assume arbitrarily low values and the coupling factors (and the global mass ratios) assume high values. In this connection one is reminded that undulations in the non-average values in the response quantities of a complex dynamic system are suppressed only when $b_s(\omega)$ exceeds unity. [cf. Appendix B.] However, the absence or the presence of undulations in that non-average response do not bear on the criterion for the validity of (SEA); (SEA) is validated as long as the modal overlap parameter $b_s(\omega)$ exceeds the corresponding minimum value of

this parameter; $b_s(\omega)>[b_s(\omega)]_M$. Indeed, the employment of (SEA) may be valid for values of both $[b_s(\omega)]_M$ and $b_s(\omega)$ that may be less than unity. Equation (38) makes clear that the validity of (SEA) merely requires that $b_s(\omega)$ exceeds $[b_s(\omega)]_M$. The validity does not demand that undulations be absent in the non-average values of the response quantities. [cf. Appendix B.] Of course (SEA) yields largely average response quantities; which are commensurably averaged a la Skudrzyk [3, 14].

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Appendix A. Discrete and Continuous Distributions

The expression for the resonance frequency distribution is formally stated. In the master dynamic system the expression is

$$(\omega_n/\omega_o) = [\omega(n)/\omega_o] = X_o(n) \qquad , \text{(A1a)}$$

and in the adjunct dynamic system the expression is

$$(\omega_r/\omega_o) = [\omega(r)/\omega_o] = X_s(r) \qquad , \text{(A1b)}$$

where (n) and (r) designate the (n)th and the (r)th *modes* that reside within the frequency bandwidth $(\Delta\omega)$ centered about the frequency (ω_o), in the master and in the adjunct dynamic systems, respectively, and $X_o(n)$ and $X_s(r)$ are the corresponding modal distributions in the frequency band defined by $\{\Delta\omega, \omega_o\}$ [3, 13, 15, 16]. In Eq. (A1) the quantities and parameters are stated in a manner that allows for a conceptual smear from the discrete-to-the continuous in the frequency band $\{\Delta\omega, \omega_o\}$. As an example, the quantities $X_o(n)$ and $X_s(r)$ are chosen to be in the forms

$$X_o(n) = [1 + \{(1+N_o) - 2n\}(\gamma_o/2N_o)]^{-1} \;\; ; \quad \gamma_o = 0.3 \;\; ; \quad N_o(\omega) \geq 1 \qquad , \text{(A2a)}$$

$$X_s(r)=\left[1+\{(1+N_s)-2r\}(\gamma/2N_s)\right]^{-1/2} \;\; ; \quad \gamma=0.6 \;\; ; \quad N_s(\omega)\geq 1 \qquad . \text{(A2b)}$$

so that they reasonably span the same frequency band $\{\Delta\omega,\omega_o\}$ [13,15]. [cf. Fig. 4a.] In this connection, one may further define from Eqs. (A1a) and (A1b) the loss factor distributions in the master and adjunct dynamic systems

$$\eta_n=\eta_o(n)=b_o(n)[X_o(n)]^{-1}[\partial X_o(n)/\partial n] \qquad , \text{(A3a)}$$

and

$$\eta_r=\eta_s(r)=b_s(r)[X_s(r)]^{-1}\;[\partial X_s(r)/\partial r] \qquad , \text{(A3b)}$$

respectively, where $b_o(n)$ and $b_s(r)$ are the respective local modal overlap parameters. From Eqs. (A2) and (A3) one obtains

$$\eta_o(n)=b_o(n)[X_o(n)](\gamma_o/N_o) \; ; \; b_o(\omega)\Rightarrow b_o(\omega)(\omega/\omega_o)[\gamma_o/N_o(\omega)] \qquad , \text{(A4a)}$$

$$\eta_s(r)=b_s(r)[X_s(r)]^2(\gamma/2N_s) \; ; \; \eta_s(\omega)\Rightarrow b_s(\omega)(\omega/\omega_o)^2[\gamma/2N_s(\omega)] \quad , \text{(A4b)}$$

where the conversion from the index-domain into the frequency-domain is governed by

$$(\omega/\omega_o) \Rightarrow X_o(n) \;\; ; \;\; (\omega/\omega_o) \Rightarrow X_s(r) \qquad , \text{(A5)}$$

and it is observed that the frequency bandwidths are defined by the values of $X_o(n)$ and $X_s(r)$ at the end-points; namely

$$X_o(1/2) = [1 + (\gamma_o/2)]^{-1} \;\; ; \;\; X_o\{N_o + (1/2)\} = [1 - (\gamma_o/2)]^{-1} \qquad , \text{(A6a)}$$

$$X_s(1/2) = [1 + (\gamma/2)]^{-1/2} \;\; ; \;\; X_s\{N_s + (1/2)\} = [1 - (\gamma/2)]^{-1/2} \qquad . \text{(A6b)}$$

[cf. Fig. 4b.] These limits are commensurate with a frequency bandwidth $\Delta(\omega)$ in the form

$$[\Delta(\omega)/\omega_o] \cong \gamma_o \cong (\gamma/2); \;\; 2\gamma_o \cong \gamma \cong 0.6 \qquad . \text{(A7)}$$

On the other hand, the values of the average loss factors (η_o) and (η_s) are estimated from Eqs. (A3) and (A6) to be

$$\eta_o = (N_o)^{-1} \sum_n^{N_o} \eta_o(n) \Rightarrow (N_o)^{-1} \int_{(1/2)}^{\{N_o+(1/2)\}} \eta_o(n) dn \quad ;$$

$$\eta_o = <\eta_o(n)> = (N_o)^{-1} \int b_o(n) [X_o(n)]^{-1} dX_o(n) \cong b_o(\gamma_o/N_o) \qquad , \text{(A8a)}$$

$$\eta_s = (N_s)^{-1} \sum_r^{N_s} \eta_s(r) \Rightarrow (N_s)^{-1} \int_{(1/2)}^{\{N_s+(1/2)\}} \eta_s(r) dr \quad ;$$

$$\eta_s = <\eta_s(r)> = (N_s)^{-1} \int b_s(r) [X_s(r)]^{-1} dX_s(r) \cong b_s(\gamma_s / 2N_s) \qquad . \text{(A8b)}$$

The replacement of a summation by an integration, as is stated cavalierly in Eq. (8), needs to be properly qualified. The extrapolations and the interpolations of the summand to establish the integrand, as the indices (r) and/or (n) are given continuous forms, must render the latter a fairly smooth function in the continuous variables (r) and/or (n), respectively. This smoothing of the integrand does not only facilitate the replacement of the summations by integrations; e.g., as exemplified in Eq. (8), but this smoothing is an averaging process similar to the mean-value averaging proposed by Skudrzyk [14]. To illustrate this process and render it more concrete one may proceed to analyze Eq. (A8b) in more detail. Concentrating on the first of this equation, one may, by extrapolations and interpolations, insert a step in the form

$$\eta_s = (N_s)^{-1} \sum_r^{N_s} \eta_s(r) \cong (N_s)^{-1} (N'_s)^{-1} \sum_{r=1}^{N_s} \sum_{r' = (-N'_s/2)}^{(N'_s/2)} \eta_s[r + (r'/N'_s)] \qquad , \text{(A9a)}$$

where (r') is a running index and N_s' is as large compared with unity as is suitable to render the summand, in this equation, smooth as a function of $[r+(r'/N_s')]$ with (r) given one of its sequential integer values. Then, in the limit, if the summations in Eq. (A9a) are absolutely convergent (not conditionally convergent) one may state the equivalence

$$(N_s)^{-1}(N_s^1)^{-1}\sum_{r=1}^{N_S}\sum_{r'=(N_S'/2)}^{(N_S'/2)}\eta_s[r+(r'/N_s')]$$

$$\Rightarrow (N_s)^{-1}\int_{(1/2)}^{\{N_S+(1/2)\}}\eta_s(r)dr \qquad , \text{(A9b)}$$

where the index (r) in the integral is piece-wise continuous with a grain that is determined by the largeness of (N_s') compared with unity. Clearly, (N_s') may be set as high as smoothing may require and beyond. In the limit the summand, in the sum over (r) from r=1 to r=N_s, is smooth and the summation qualifies for replacement by integration. Notwithstanding that were the indices (r) and/or (n) render a priori continuous in the summand, would have resulted in an integrand that is smooth, so that the replacement of the summation by an integration would be naturally qualified [6]. It is provided that within the limits: $1\le r\le N_s$ and $1\le n\le N_o$ the modal overlap parameters $b_s(r)$ and $b_o(n)$

and the resonance frequency distributions $X_s(r)$ and $X_o(n)$ are reasonably smooth functions of the continuous (r) and (n), respectively; e.g., as represented in Fig. 4. The replacing of the summations by the integrations in Eq. (8) is then naturally qualified.

Attention is now focused on the induced loss factor. It is further developed that the discrete expression for the induced loss factor $\eta_I^d(n)$ and its continuous counterpart $\eta_I(n)$, both pertaining to the (n)th mode in the master dynamic system, may be cast, respectively, in the forms

$$\eta_I^d(n) = \sum_r^{N_S} \eta_I(n,r) \; ; \; \eta_I(n) = (N_S')^{-1} \sum_r^{N_S} \sum_{r'=(-N_S'/2)}^{(N_S'/2)} \eta_I[n, r+(r'/N')] \quad , \text{(A10a)}$$

and their averages, over the modes in the master dynamic system, are given by

$$\eta_I^d = <\eta_I^d(n)> = (N_o)^{-1} \sum_n^{N_O} \eta_I^d(n) \; ;$$

$$\eta_I = (N_o)^{-1} (N_o')^{-1} \sum_n^{N_O} \sum_{n'=(-N_O'/2)}^{(N_O'/2)} \eta_I[n+(n'/N_o')] \quad . \text{(A11a)}$$

[cf. Eq. (A9).] If (N_s') and (N_o') are assigned large enough values and the extrapolations and interpolations are properly chosen to render the summand smooth, the summations may qualify to be replaced by integrations. Then, the values of the average induced loss factors $\eta_I(n)$ and η_I are estimated, from Eqs. (A10a) and (A11a), to be

$$\eta_I(n) \Rightarrow \int_{(1/2)}^{\{N_s+(1/2)\}} \eta_I(n,r)\,dr \quad , \text{(A10b)}$$

$$\eta_I \Rightarrow (N_o)^{-1} \int_{(1/2)}^{\{N_o+(1/2)\}} \int_{(1/2)}^{\{N_s+(1/2)\}} \eta_I(n,r)\,dr\,dn \quad . \text{(A11b)}$$

Again, the replacement of a summation by an integration, as stated in Eqs. (A10) and (A11), is thus procedurally qualified. [cf. Eq. (A9).]

Once again, it may be advantageous to institute a transformation of variables in Eqs. (A10b) and (A11b); namely

$$\eta_I(n) \Rightarrow \int \eta_I(n,r)[f_s(r)]\,dX_s(r) \quad , \text{(A10c)}$$

$$\eta_I \Rightarrow (N_o)^{-1} \iint \eta_I(n,r)[f_s(r) f_o(n)] dX_s(r) dX_o(n)$$, (A11c)

where the limits on the integrals over $X_s(r)$ and/or over $X_o(n)$ are conveniently omitted, but are obviously those expressed in Eq. (A6) and

$$f_o(n) = \{dX_o(n)/dn\}^{-1} \quad ; \quad f_s(r) = \{dX_s(r)/dr\}^{-1}$$. (A12)

Thus, the utilization of extrapolations and interpolations in the procedure to replace the summations, in Eqs. (A10a) and (A11a), by the integrations, in Eqs. (A10c) and (A11c), entails rendering the integrands in these integrations fairly smooth functions of the continuous forms of $X_s(r)$ and/or $X_o(n)$, respectively. [cf. Fig. 4a and Eq. (A9b).] The assurance that this smoothness can be implemented, with respect to $f_s(r)$ and $f_o(n)$, is then readily made. For example, from Eqs. (A2) and (A12) one obtains

$$f_o(n) = (N_o/\gamma_o)\{X_o(n)\}^{-2}$$, (A13a)

$$f_s(r) = (2N_s/\gamma)\{X_s(r)\}^{-3}$$, (A13b)

which are reasonably smooth within the chosen frequency bandwidth indicated in Eqs. (A6) and (A7). It is to be understood that, in Eqs. (A10c) and (A11c), where appropriate, the vector-variable $\{n,r\}$ needs to be replaced by $\{X_o(n), X_s(r)\}$. Here the assumption is that these two two-vectors are properly related, whether they lie in the discrete domain or in the continuous domain. [cf. Eq. (A5).] Unfortunately, without further investigation no assurance of smoothness can be readily given with respect to the integrand factor $\eta_I(n,r)$ in Eqs. (A10c) and (A11c). The smoothness or lack of smoothness in $\eta_I(n,r)$ is examined in the next appendix; i.e., in Appendix B.

Appendix B Smoothing of the Induced Loss Factor $\eta_I(n,r)$

The equation of motion that governs the induced modal loss factor $\eta_I(n,r)$, between the (n)th mode in the master dynamic system and the (r)th mode in the adjunct dynamic system, may be expressed in the form

$$\eta_I(n,r) = -\mathrm{Im}\{(y)^2(m_r/m_n)\,[\{1-(z_r)^2(1+i\eta_r)\}\,\{\mathrm{m}_{nr}-(z_{nr})^2(1+i\eta_{nr})\}-(q_{nr}/y)^2]\,[(1+\mathrm{m}_{nr})-(z_{nrr})^2(1+i\eta_{nrr})]^{-1}\} \quad \text{, (B1a)}$$

where a similarity condition is imposed in the form

$$(z_r)^2 = \alpha_r\{X_s(r)/y\}^2 \; ; \; (z_{nr})^2 = \alpha_{nr}\{X_s(r)/y\}^2 \; ; \; (z_{nrr})^2 = (\alpha_r + \alpha_{nr})\{X_s(r)/y\}^2 \; ;$$
$$y = (\omega/\omega_o) \quad ; \quad (z_{nrr})^2(1+i\eta_{nrr}) = (z_r)^2(1+i\eta_r) + (z_{nr})^2(1+i\eta_{nr}) \quad \text{, (B2a)}$$

and the coupling parameters (m_{nr}) and (q_{nr}) are

$$\mathrm{m}_{nr} = (m_{nr}/m_r) \quad ; \quad (q_{nr})^2 = 4\mathrm{m}_{nr}(z_{nr})^2(1+i\eta_{nr}) + (g_{nr}/y)^2 \quad \text{. (B2b)}$$

The parameters that describe the adjunct dynamic system - - mass-stiffness-dashpot - - are (m_r), $\{\alpha_r, X_s(r)\}$ and (η_r) and those that describe the coupling between the master and the adjunct dynamic

systems -- mass-stiffness-dashpot-gyroscopic -- are (m_{nr}), $\{\alpha_{nr}, X_s(r)\}$, (η_{nr}) and (g_{nr}). The similarity imposed on the stiffness associated with the mass (m_r), both with respect to the (r)th mode and with respect to the coupling of that mode to the (n)th mode in the master dynamic system, allows one to incorporate the couplings in the description of the adjunct dynamic system. This incorporation is a useful interpretative ploy [13, 15]. The resonant nature of $\eta_I(n,r)$, stated in Eq. (B1) is obvious. The modes are defined in the (z_{nrr})-domain by requiring the quantity $[(1+m_{nr})-(z_{nrr})^2]$ to vanish. Then the width of the mode in the (z_{nrr})-domain is determined by $[(1+m_{nr})\eta_{nrr}]$. As long as the local modal overlap parameters (b_{nrr}) are below unity, the modes are distinguishable and the distribution of $\eta_I(n,r)$, for discrete values of the index (r) and a fixed value of the index (n), is not smooth as a function of (ω/ω_o) in $(\Delta\omega/\omega_o)$. The presence of modes in $\eta_I(n,r)$, as a function of (ω/ω_o) in $(\Delta\omega/\omega_o)$, for discrete (r) and (n), is readily discernible. Indeed, the presence of modes in the summand $\eta_I(n,r)$, in this case, is even discernible in the summ $\eta_I^d(n)$, stated in the first of Eq. (A10a). The modal

character of $\eta_I^d(n)$, for the complex depicted in Fig. 3, as a function of (ω/ω_o) in $(\Delta\omega/\omega_o)$ and for a modal overlap parameter $b_{nrr} \cong b_s = 0.1$, is exemplified in Fig. 9a. The modal character in $\eta_I^d(1); n=1$, is clearly demonstrated in this figure. It emerges that were the modal overlap parameters (b_{nrr}) above unity, the modes are no longer distinguishable and the values of $\eta_I^d(n)$, for discrete values of the index (r) and a fixed value of the index (n), is reasonably smooth as a function of (ω/ω_o) in $(\Delta\omega/\omega_o)$. The presence of modes in $\eta_I^d(n)$, as a function of (ω/ω_o) in $(\Delta\omega/\omega_o)$, is obscured. Indeed, the non-modal (smooth) character of $\eta_I^d(n)$, for the complex depicted in Fig. 3; namely for $\eta_I^d(1)$ and for a modal overlap parameter $b_{nrr} \cong b_s = 2.0$, is exemplified in Fig. 9b. The absence of modal character in $\eta_I^d(1)$ in Fig. 9b, is, thus, clearly demonstrated. [cf. Fig. 9a.] Moreover, it follows that for modal overlap parameters that exceed unity; $b_s > 1.0$, the replacement of the summation in Eq. (A10a) by the integration in Eq. (A10b) is naturally qualified. What happens if the modal overlap parameters are below unity; namely, when $b_s \leq 1.0$? It is

argued that for discrete values of the index (r) and a fixed value of the index (n), $\eta_I(n,r)$ is not a smooth summand and, therefore, in this case, the replacement of a summation by an integration is not naturally qualified. However, if the index (r) is made into a continuous variable, this action, as the second of Eq. (A10a) suggests, is tantamount to planting multiplicity of modes in between the originally discrete modes. These generated modes, as do the initial modes, carry modal extents that are largely determined by loss factors of values given by $[(1+m_{nr})\eta_{nrr}]$. Clearly, the increase in the modal density due to this transition from the discrete-to-the continuous (r) renders $\eta_I(n,r)$ smooth and, therefore, qualifying the replacement of the summation by integration. In this procedure the artificial increase in the modal density is, however, compensated by averaging over the number of added modes, as exemplified in Eqs. (A9a) - (A11a). The result of replacing a summation by an integration is commensurate with an artificial increase in the modal overlap parameter to values that subdue modal recognitions. Indeed, the a priori replacement of a summation by an integration results in values of the induced loss factor from $\eta_I^d(n)$ to $\eta_I(n)$ that match these obtained by the mean-value averaging proposed by Skudrzyk [14]. [cf. Eq. (A10a).]

In EA: (Master in isolation)

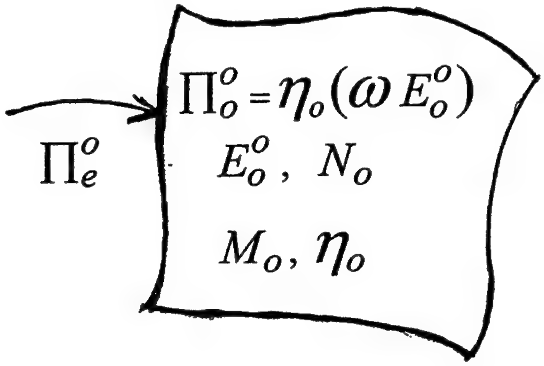


Fig. 1. An externally force-driven isolated master dynamic system.

Π_e^o = input power generated by an external force-drive.

E_o^o = stored energy = $N_o \varepsilon_o^o$; ε_o^o = modal stored energy.

N_o = number of modes = $\Delta(\omega)\nu_o$; ν_o = modal density.

M_o = global mass = $N_o m_o$; m_o = modal mass.

η_o = indigenous loss factor; ω = frequency variable.

$\Delta(\omega)$ = frequency bandwidth.

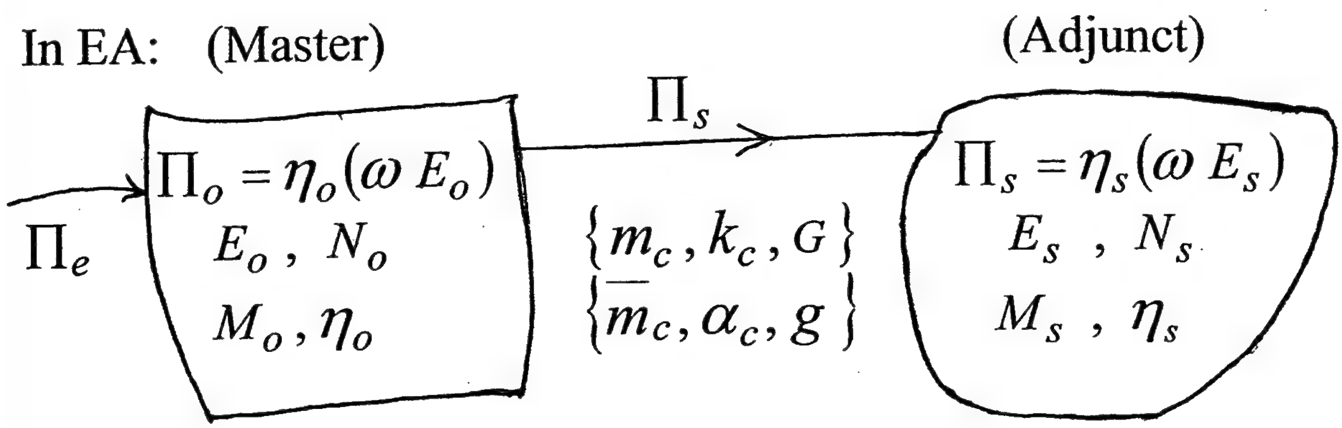


Fig. 2. An externally force-driven master dynamic system passively coupled to an adjunct dynamic system.

Subscript (o) designates quantities and parameters that pertain to the master dynamic system.

Subscript (s) designates quantities and parameters that pertain to the adjunct dynamic system.

Π_s = the net power that is imparted to the adjunct dynamic system from the master dynamic system.

The vector $\{m_c, k_c, G\}$ and its normalized form $\{\overline{m}_c, \alpha_c, g\}$ describe the coupling coefficients {mass, stiffness and gyroscopic} between the two dynamic systems. [cf. Fig. 1.]

Π_e = input power generated by an external force-drive.

E_o = stored energy in the master dynamic system.

E_s = stored energy in the adjunct dynamic system.

N_s = number of modes in adjunct dynamic system = $\Delta(\omega)\nu_s$;

ν_s = modal density in adjunct dynamic system.

η_s = indigenous loss factor in the adjunct dynamic system.

M_s = global mass of the adjunct dynamic system = $N_s m_s$;

m_s = modal mass in the adjunct dynamic system.

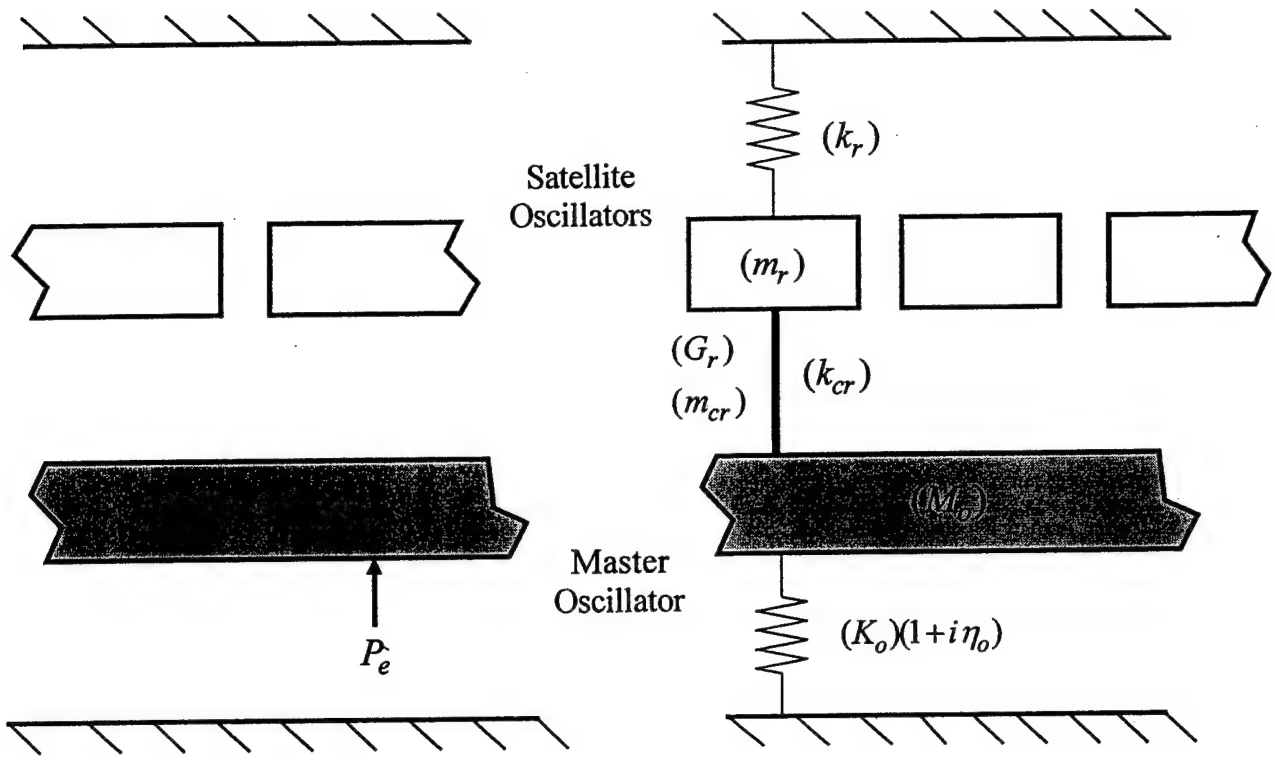


Fig. 3. A Sketch of a specific complex dynamic system comprising a master dynamic system consisting of a single harmonic oscillator (a master oscillator) and an adjunct dynamic system consisting of several harmonic (satellite) oscillators. The satellite oscillators are identified by the index (r). The satellite oscillators are uncoupled to each other. Also, only the master oscillator is externally force-driven.

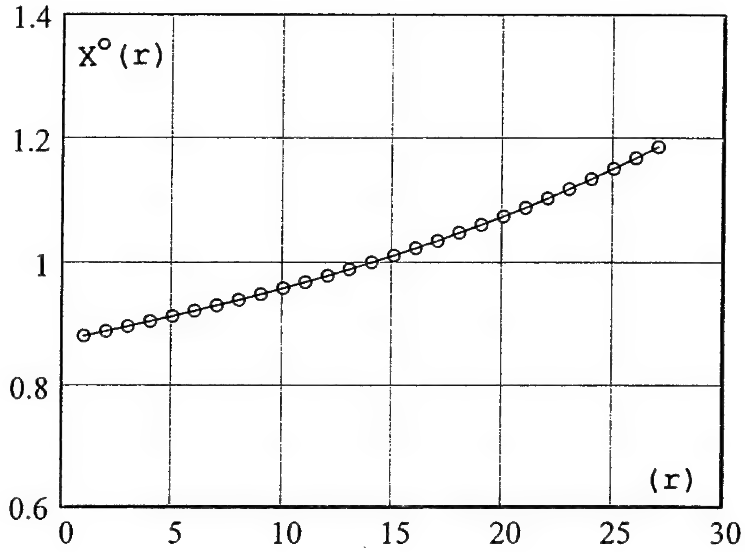


Fig. 4a. The normalized resonance frequency distribution $X^{o}(r)$ of the harmonic oscillators in the adjunct dynamic system (the satellite oscillators) as a function of the index (r). $X^{o}(r)$ is as stated in Eq. (29a) and $N_s = 27$. O Discrete, —— Continuous.

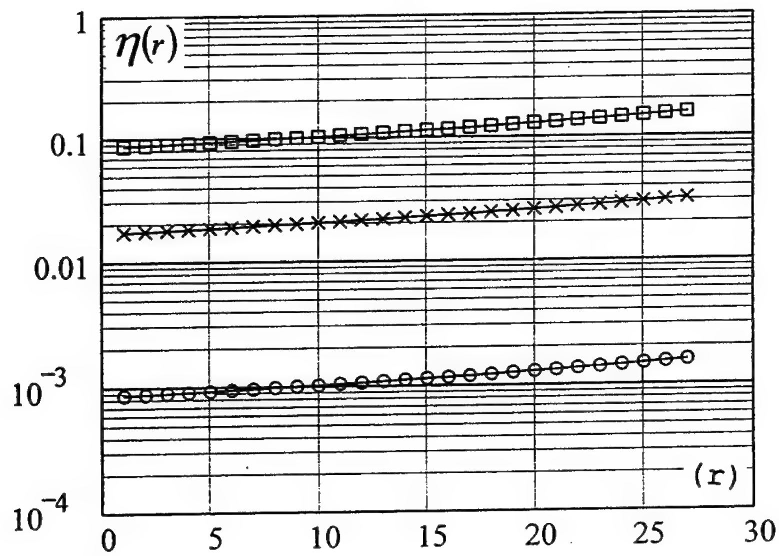


Fig. 4b. The localized loss factor $\eta(r)$ of the harmonic oscillator in the adjunct dynamic system (the satellite oscillators) as a function of the index (r). $\eta(r)$ as stated in Eq. (29b), $N_s = 27$ and

o Discrete, —— Continuous for $b = 0.1$,

x Discrete, —— Continuous for $b = 2.0$,

□ Discrete, —— Continuous for $b = 10$.

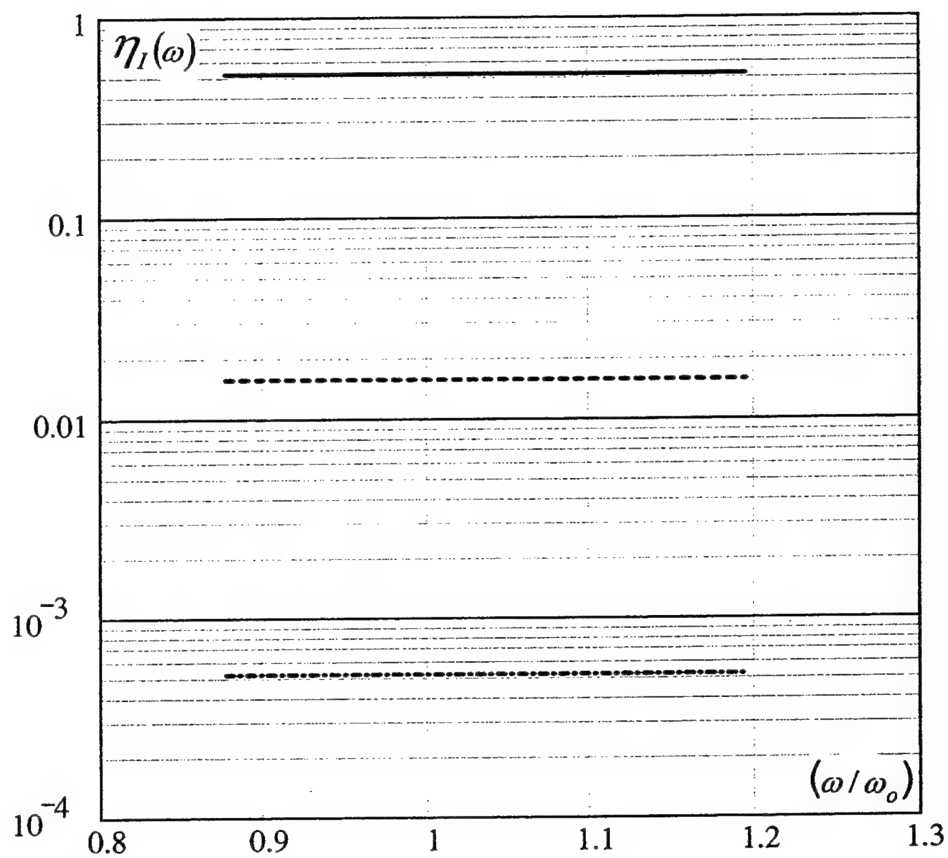


Fig. 5. The induced loss factor $\eta_I(\omega)$ as a function of the normalized frequency (ω/ω_o) for $N_s = 27$, $(M_s/M_o) = 0.1$ and for three values of the coupling factor $C(\omega)$; $C(\omega) = 1.0$ (solid curve), $C(\omega) = 3x10^{-2}$ (dash curve), and $C(\omega) = 10^{-3}$ (dash-dot curve). The frequency bandwidth $[\Delta(\omega)/\omega_o] = (\gamma/2) \cong 0.3$.

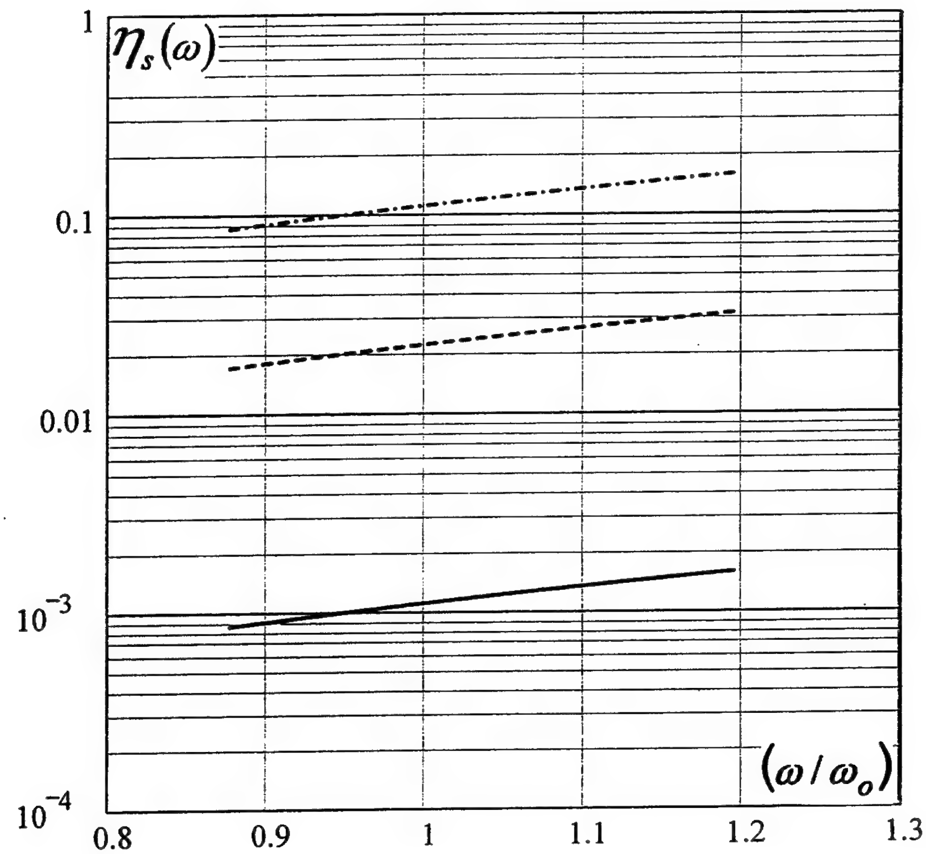


Fig. 6. The indigenous loss factor $\eta_s(\omega)$ in the adjunct dynamic system as a function of the normalized frequency (ω/ω_o) for $N_s = 27, (M_s/M_o) = 0.1$ and for three values of the modal overlap parameter $b_s(\omega)$; $b_s(\omega) = 0.1$ (solid), $b_s(\omega) = 2.0$ (dash) and $b_s(\omega) = 10.0$ (dash-dot). The frequency bandwidth $[\Delta(\omega)/\omega_o] = (\gamma/2) \cong 0.3$.

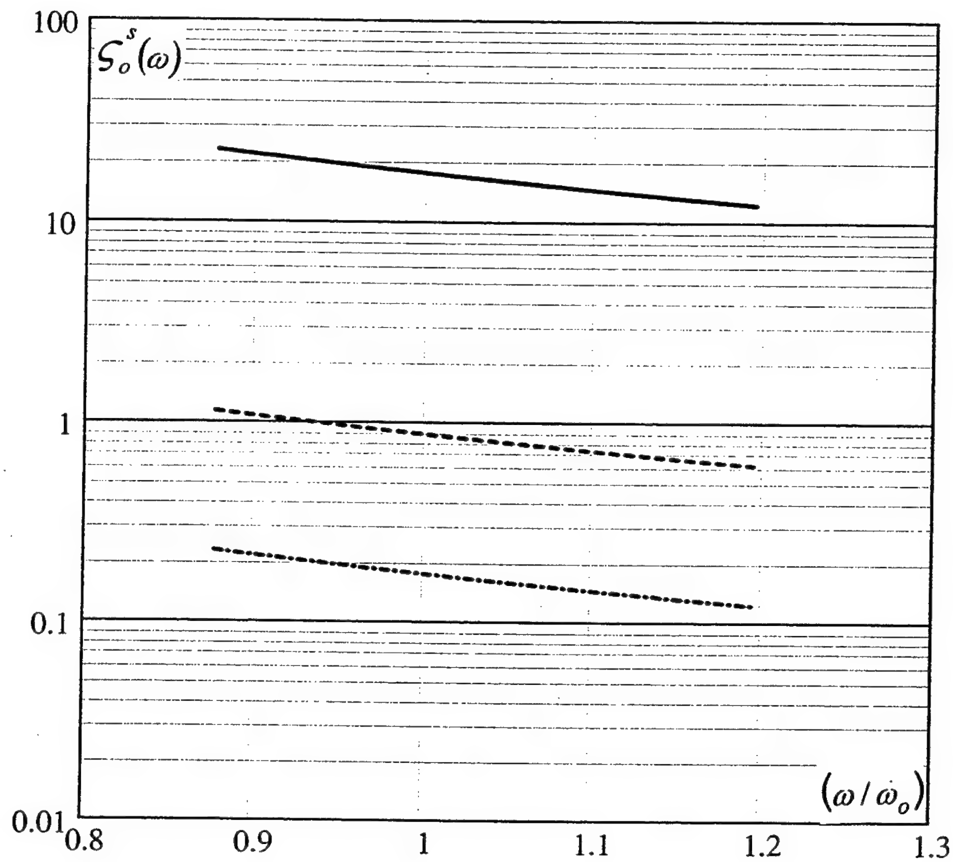


Fig. 7a. The modal coupling strengths $\varsigma_o^s(\omega)$ as a function of the normalized frequency (ω/ω_o) for a coupling factor $C(\omega)=1.0,\ N_s=27,\ (M_s/M_o)=0.1$ and for three values of the modal overlap parameter $b_s(\omega)$; $b_s(\omega)=0.1$ (solid curve), $b_s(\omega)=2.0$ (dash curve) and $b_s(\omega)=10.0$ (dash-dot curve).

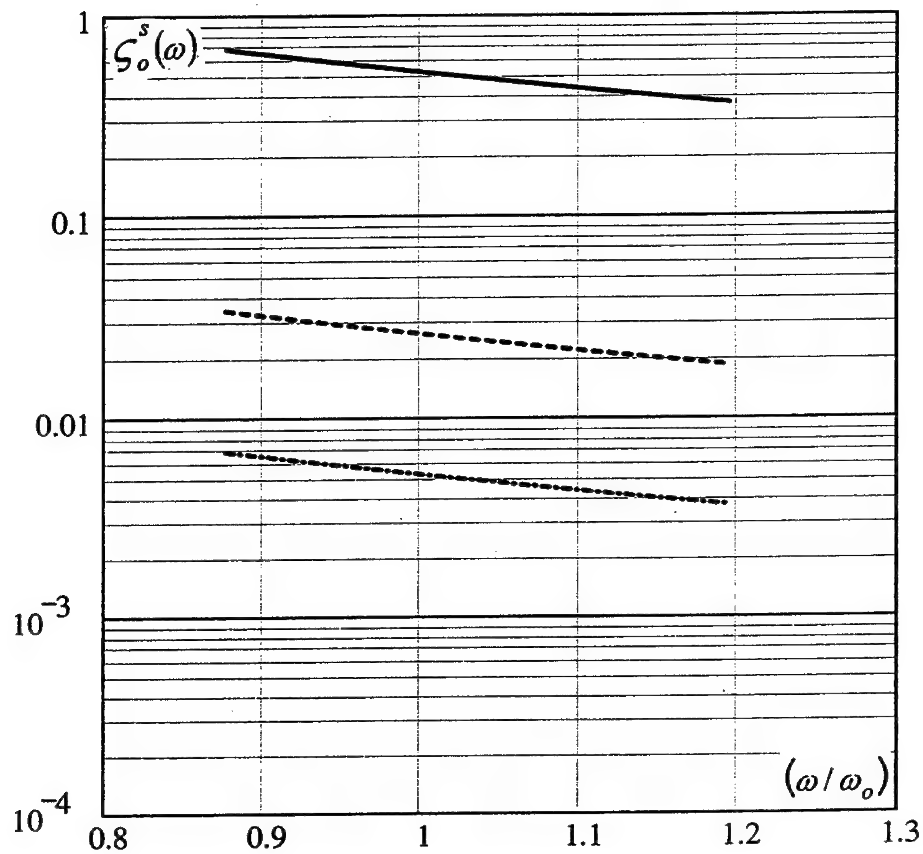


Fig. 7b. The modal coupling strengths $\varsigma_o^s(\omega)$ as a function of the normalized frequency (ω/ω_o) for a coupling factor $C(\omega)=3x10^{-2}$, $N_s=27$, $(M_s/M_o)=0.1$ and for three values of the modal overlap parameter $b_s(\omega)$; $b_s(\omega)=0.1$ (solid curve), $b_s(\omega)=2.0$ (dash curve) and $b_s(\omega)=10.0$ (dash-dot curve).

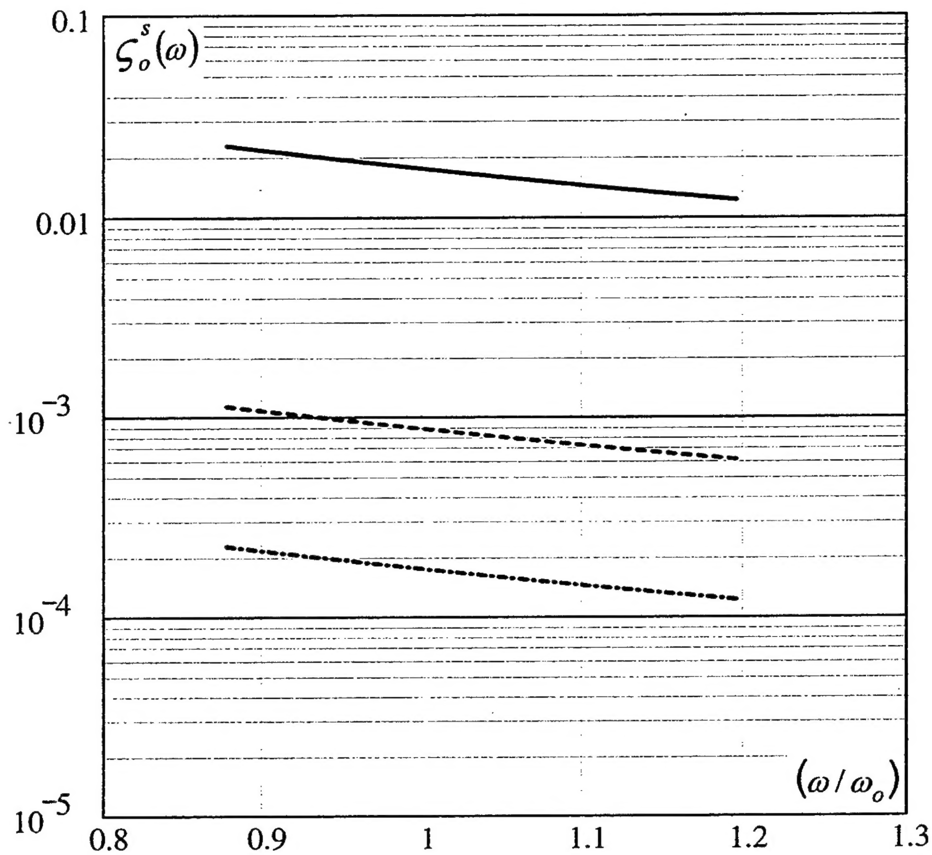


Fig. 7c. The modal coupling strengths $\varsigma_o^s(\omega)$ as a function of the normalized frequency (ω/ω_o) for a coupling factor $C(\omega)=10^{-3}$, $N_s=27$, $(M_s/M_o)=0.1$ and for three values of the modal overlap parameter $b_s(\omega)$; $b_s(\omega)=0.1$ (solid curve), $b_s(\omega)=2.0$ (dash curve) and $b_s(\omega)=10.0$ (dash-dot curve).

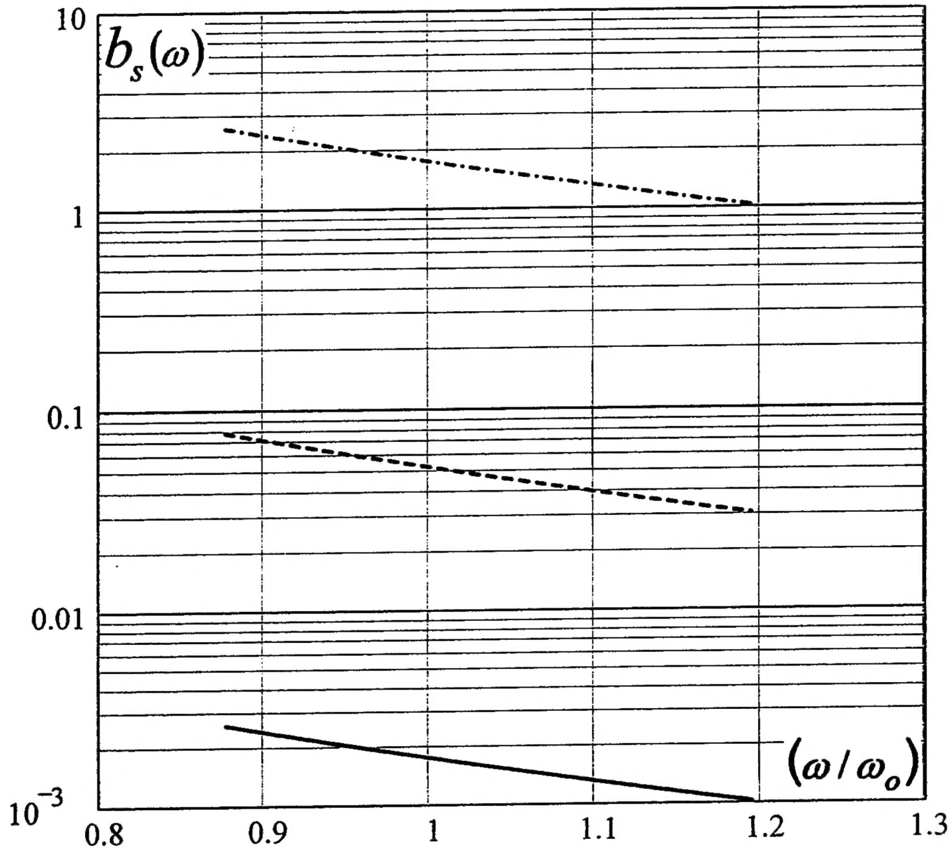
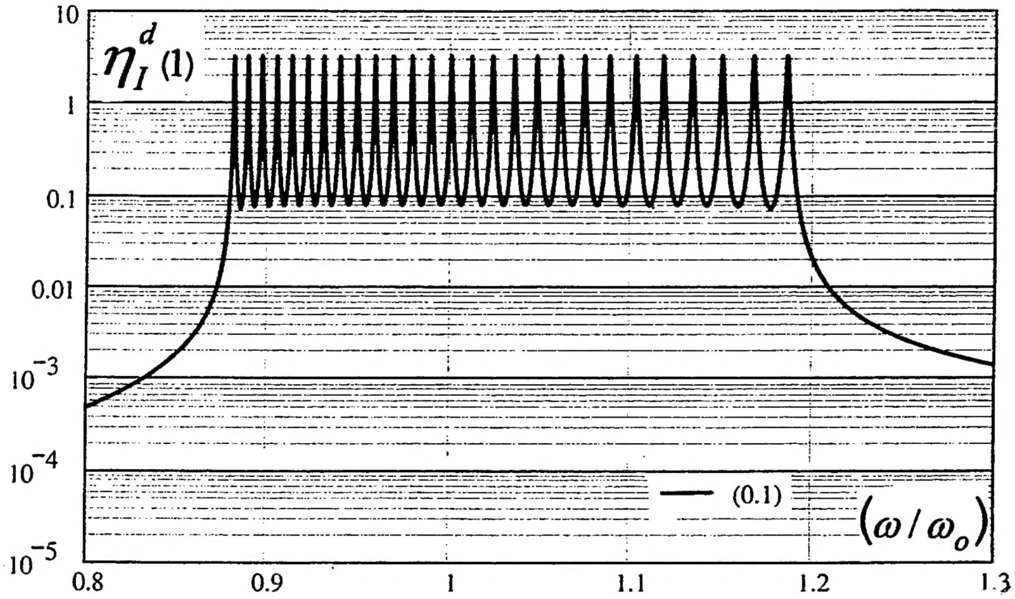


Fig. 8. The minimum value of the modal overlap parameter $[b_s(\omega)]_M$, as a function of the normalized frequency (ω / ω_o), for $N_s = 27$ and $(M_s / M_o) = 0.1$ and for three values of the coupling factor $C(\omega)$; $C(\omega) = 1$ (dash-dot), $C(\omega) = 3x10^{-2}$ (dash) and $C(\omega) = 10^{-3}$ (solid). The statistical energy analysis is *sufficiently* valid for corresponding values of the modal overlap parameter $b_s(\omega)$ that exceed the minimum value shown in this figure.

a.



b.

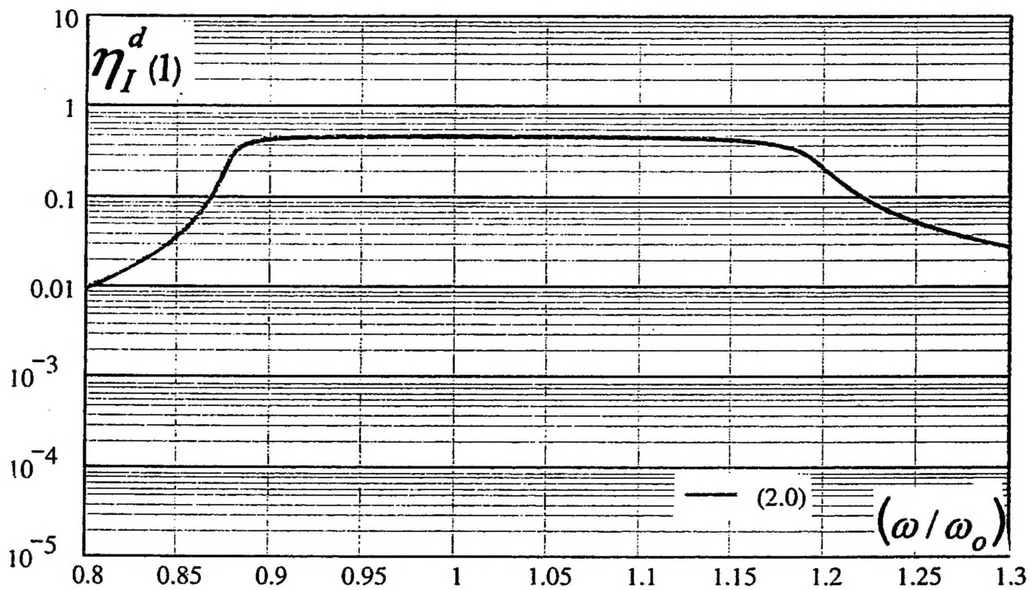


Fig. 9. Discrete induced loss factor $\eta_I^d(1)$ as a function of (ω/ω_o), for a stiffness control coupling form with $\alpha_c = 1.0[\alpha = 0.0.]$, $g = \overline{m}_c = 0$ [Sprung masses]. The modal distribution in the adjunct dynamic system is as stated in Eq. (29a) and $b(r)$ in Eq. (29b) is given by b.

a. With $b = (0.1)$ b. With $b = (2.0)$

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